

HW/due (today + 1)

day 54

4.1/ 68-70, 76, 78

4.2/ 41-44, 49, 53, 57-60 AND 3.11/31-33

4.3/ 1-8

HW/ due after r

4.1/ 80-81

4.2/ 54, 61-62, 71-74, 83

4.3/ 9-10, 15-16, 21-24

HW/ due later but not
too late

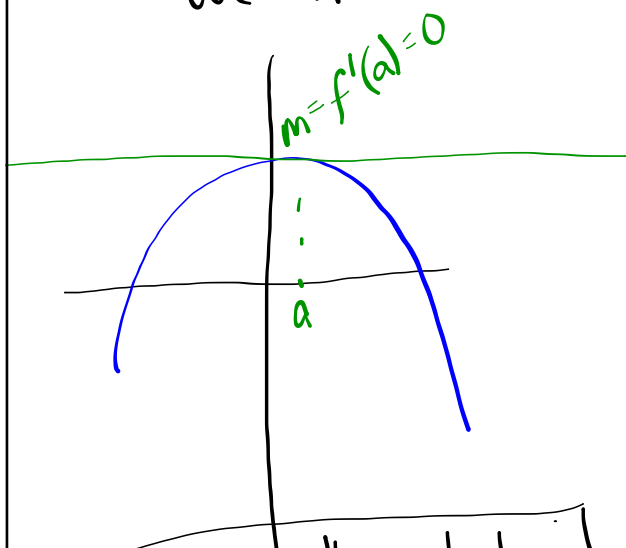
4.1/ 79, 82

4.2/ 84-87

4.3/ 25-27, 37,
43-45, 50

Another use for concavity (and the second derivative) day 54

When the second derivative exists (and \therefore the first derivative exists and \therefore the original function \dots)
we have this situation for a local maximum...



If $f(x)$ has a horizontal tangent at $x=a$, AND if $f(x)$ is concave down [i.e. $f''(a) < 0$]
 $f''(a)$ is NEG]
THEN $f(x)$ has a relative maximum at $x=a$

i.e. "id est" \equiv that is
e.g. "exempli gratia" \equiv for example
Latin you should know

day 54

You are being trained as
function doctors

$f(x)$ "Dr Tiff"
- horizontal
- 2 vertical asymptotes

$f'(x)$ * $f'(x)=0$, $f'(x)$ undefined

Why? we could have a maximum
or minimum there

first derivative
test

essentially = ... create
a sign chart
of $f'(x)$

before
critical pt

crit
pt

= 0,
und

after
crit
pt

intervals
of $f(x)$
increasing
+
decreasing

Second derivative
test

find the second
derivative
AT THE CRIT. PT.

$f'(a)=0$

$f''(a)$	diagnosis
negative	rel max
positive	rel min
0	IDK

pos	rel max	neg
pos	hor tang	pos
neg		neg
neg	rel min	pos

Dr Tiff, episode 2

day 54

diagnosing from $f''(x)$ * create a sign chart of $f''(x)$

$$f''(x) \quad \begin{array}{ccccccc} & + & + & 0 & - & - & 0 & + & + \\ & + & + & | & - & - & | & + & + \end{array}$$
 \Rightarrow intervals where the f'' is concave up or concave down

inc vs dec

inc

inc

dec

dec

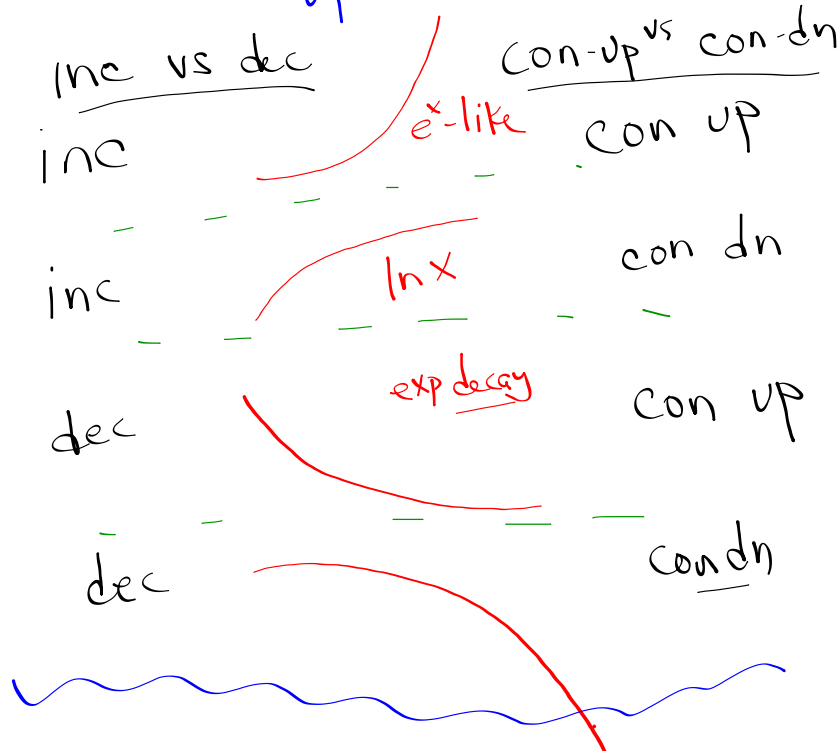
con-up vs con-dn

con up

con dn

con up

con dn



A point of inflection is where concavity changes.

Pt of inflection: candidates are:

where $f''(x) = 0$

OR

 $f''(x)$ undhow do I determine YAY or NAY?
sign chart of $f''(x)$
check surrounding $f''(x)$ for pos/neg

day 54

4.1/40) $f(x) = \frac{x}{(x^2+3)^2}$ on $[-2, 2]$

crit pt
abs extr
graph

$$\begin{aligned} a) f'(x) &= \frac{\frac{d}{dx}(x) \cdot (x^2+3)^2 - x \frac{d}{dx}(x^2+3)^2}{((x^2+3)^2)^2} \\ &= \frac{(x^2+3)^2 - x(2(x^2+3) \cdot \frac{d}{dx}(x^2+3))}{(x^2+3)^4} \\ &= \frac{(x^2+3)^2 - 2x(x^2+3)(2x)}{(x^2+3)^4} = \frac{(x^2+3)[x^2+3-4x^2]}{(x^2+3)^4} \end{aligned}$$

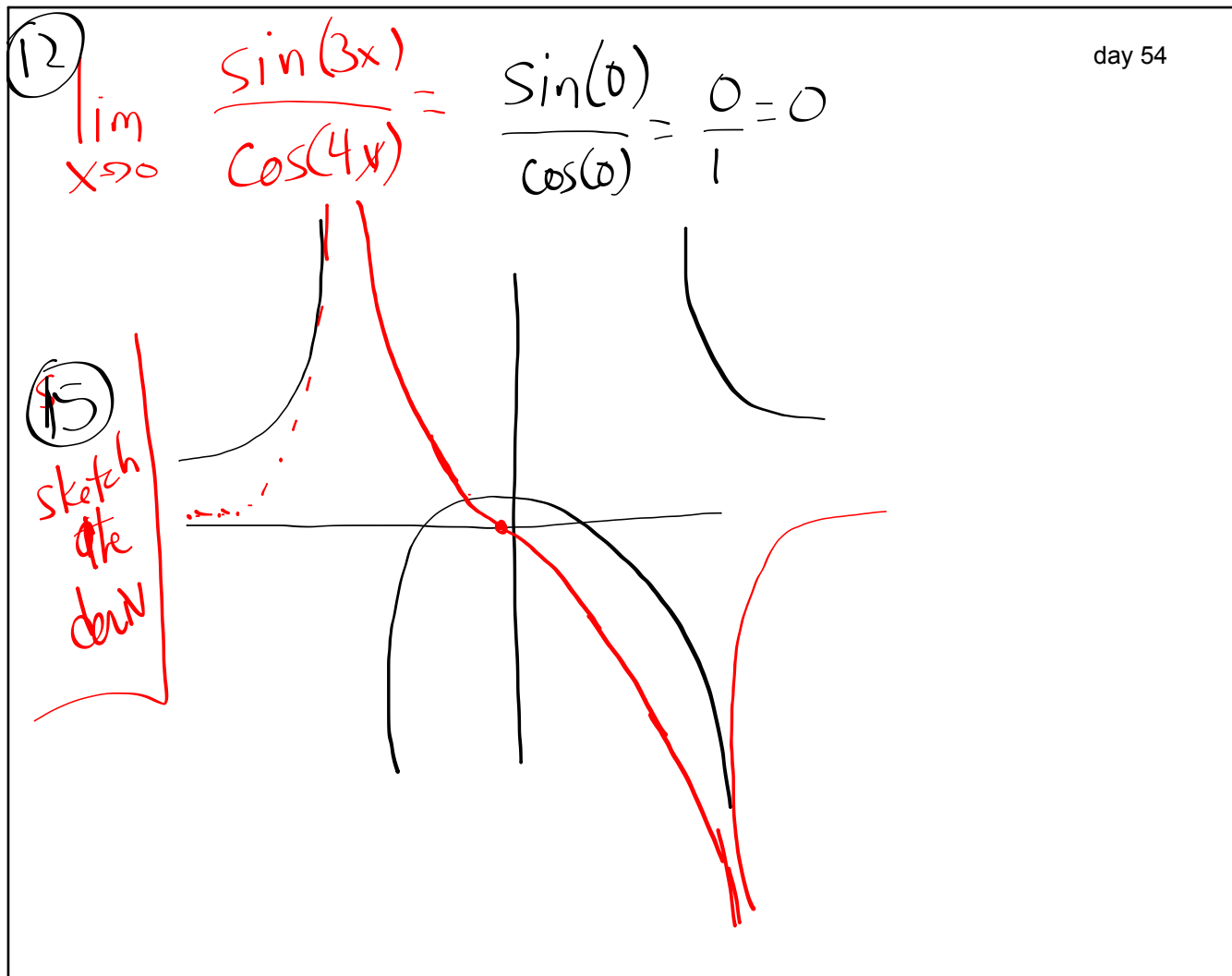
$$\begin{aligned} b) f'(x) = 0 &\Rightarrow 5x^2 + 3 = 0 \quad x = \pm \sqrt{-\frac{3}{5}} \text{ oops none} \\ f'(x) \text{ undefined} &\Rightarrow (x^2+3)^3 = 0 \Rightarrow x^2+3 = 0 \\ &\Rightarrow x = \pm \sqrt{-3} \text{ oops none} \end{aligned}$$

day 54

2	2	2	4	5	4	2	3	4	3	4	2	2	5	3	
1	2	3	4	5	6	7a	7b	8	9	10	11	12	14	15	

points
g #

$$\begin{aligned}
 & \text{supposed} \\
 & (2x+3) \\
 & 2x+3 \frac{d}{dx} (3x^2-2) \\
 & = (2x+3)(6x)
 \end{aligned}$$



14) $x(t) = -t^2 + 4t - 3, \quad 0 \leq t \leq 5$

day 54

moving left:
 $(2, 5]$

moving right:
 $[0, 2)$

speeding up: $(2, 5]$
sign of $v(t)$
= sign of $a(t)$

$$a(t) = (-2)$$

$$x'(t) = v(t) = -2t + 4$$

$$\text{moving left} \equiv v(t) < 0$$

$$\text{moving right} \equiv v(t) > 0$$

$$v(t) = 0 \Rightarrow -2t + 4 = 0$$

$$t = 2$$

